

DETERMINATION OF THE HEAT TRANSFER CAPABILITY OF LASER MIRRORS WITH COOLED CELLS

Yu. V. Zhernovyi, I. G. Odnorozhenko, D. B. Potyagailo,
and Ya. P. Romanchuk

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A mathematical model of steady-state heat transfer in a laser mirror involving cooled prismatically shaped cells has been developed. Using cooling systems with hexahedral and tetrahedral cells (by the number of side walls) as examples, the influence of the mirror illumination nonuniformity, reflector thickness, and other parameters on the effective heat-transfer coefficient and thermal head coefficient is investigated; the physical limits for heat-transfer characteristics in the case of an unlimited increase in heat transfer from the surfaces of the cell walls have been determined.

Very high requirements are imposed on mirrors for high-power laser systems: a high optical destruction threshold and small deformations of the optical surface. For these requirements to be met, reliable systems of cooling the reflecting surface are needed. Depending on the means of heat removal, different types of mirrors can be distinguished: with a cooled porous sublayer [1], with a cooled cavity, of multichannel type, with cooled cells, etc. Mirrors with cooled cells [2], which make it possible to employ the joint effect of finning and jet cooling for heat-transfer augmentation, are particularly promising for powerful laser systems.

The laws governing the hydrodynamics and heat transfer of axisymmetric and plane air jets interacting with a plane wall were considered in detail in [3, 4]. The problem of heat transfer of a liquid jet flowing past bodies with extended surfaces has been studied to a much lesser degree, in particular, when separate jets propagate in closed cells. In [5] the results of experimental study of convective heat transfer in single water-cooled cells are presented and the data on the effective heat transfer in a jet heat exchanger involving nine water cooled cells of cylindrical shape are given. The determination of the effective heat-transfer coefficient of a wall with slotted extended surfaces on the basis of a one-dimensional mathematical model of heat transfer is considered.

Based on the mathematical model of heat transfer in a jet heat exchanger with cooled prismatically shaped cells, the effective heat transfer coefficient and the thermal head coefficient have been analytically determined in the present work for the cases of the use of cells in the form of hexahedral and tetrahedral prisms with account of the nonuniform illumination of the mirror.

In Fig. 1 the general diagram of the design of a mirror with cooled cells is shown. Reflector 1 has the shape of a circular plate $0 \leq r \leq R$, $0 \leq z \leq \delta$. It is heated by a local surface heat source with a Gaussian-type flux

$$\lambda \frac{\partial T}{\partial z} \Big|_{z=0} = -q(r), \quad q(r) = Q_0 \exp(-k_0 r^2). \quad (1)$$

The walls of the cells (fins) 2 together with base 3 and reflector 1 form the closed volume needed for a reliable (in the presence of hydrostatic pressure) circulation of liquid supplied through nozzles 4. The heated heat-transfer agent is drained through holes 5.

We assume that convective heat transfer takes place on the walls of the cells and on the parts of the reflector back surface which contacts the heat-transfer agent:

$$\frac{\partial T}{\partial n} + \frac{\mu}{\lambda} (T - T_m) = 0, \quad (2)$$

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$$\frac{\partial T}{\partial z} + \frac{\mu}{\lambda} (T - T_m) = 0 \quad (6)$$

and under the condition of thermal insulation of the side surface

$$\left. \frac{\partial T}{\partial r} \right|_{r=R} = 0.$$

Taking into account the fact that in the case of thin fins the sum of the areas of the bases of the cell walls is small as compared with the area of the entire reflector S_δ , we extend condition (6) to the entire surface S_δ to approximately determine the temperature field of the reflector. In such a statement, the solution of the stationary heat conduction problem can be presented analytically as

$$v(r, z) = \sum_{n=1}^{\infty} A_n \varphi_n(z; \alpha_n) J_0(\alpha_n r), \quad (7)$$

where

$$\begin{aligned} v &= T - T_m; \quad \varphi_1(z; \alpha_1) = \varphi_1(z) = \delta - z + \lambda/\mu; \\ \varphi_n(z; \alpha_n) &= \frac{\lambda \alpha_n \operatorname{ch}[\alpha_n(\delta - z)] + \mu \operatorname{sh}[\alpha_n(\delta - z)]}{\alpha_n [\mu \operatorname{ch}(\alpha_n \delta) + \lambda \alpha_n \operatorname{sh}(\alpha_n \delta)]}, \quad n = 2, 3, \dots; \\ A_n &= \frac{2Q_0}{\lambda R^2 [J_0(\alpha_n R)]^2} \int_0^R r \exp(-k_0 r^2) J_0(\alpha_n r) dr, \quad n = 1, 2, \dots; \end{aligned}$$

$\alpha_1 = 0, \alpha_n > 0, n = 2, 3, \dots$ are the roots of the equation $J_1(R\alpha) = 0$.

The temperature distribution in the side walls of the cells will be sought with the following simplification: at the end face P_{ji} of each wall L_{ji} of the cell D_j with an ideal thermal contact with the reflector, the temperature at each point is equal to the mean integral temperature of the corresponding portion of the reflector surface S_δ

$$\bar{v}_{ji} = \frac{1}{S(P_{ji})} \iint_{P_{ji}} v dS,$$

where $S(P_{ji})$ is the surface area of the end face P_{ji} . The opposite end face of the wall L_{ji} is assumed to be thermally insulated. Taking into account the fact that the wall thickness h is much smaller than its length l , we assume that the temperature field at each vertical section, which is orthogonal to the wall, is independent of the section location. Then the temperature of the wall L_{ji} depends only on two variables x and z (Fig. 2); it is determined as the solution of the boundary-value problem

$$\begin{aligned} \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2} &= 0, \quad |x| < \frac{h}{2}, \quad \delta < z < \delta + H; \\ \frac{\partial v}{\partial x} \pm \frac{\mu}{\lambda} v &= 0, \quad x = \pm \frac{h}{2}; \quad v|_{z=\delta} = \bar{v}_{ji}, \quad \left. \frac{\partial v}{\partial z} \right|_{z=\delta+H} = 0 \end{aligned}$$

in the form

$$v_{ji}(x, z) = 2\mu\lambda\bar{v}_{ji} \sum_{k=1}^{\infty} \frac{\beta_k A(\beta_k) \operatorname{ch}[\beta_k(\delta + H - z)] \cos(\beta_k x)}{\operatorname{ch}(\beta_k H) \cos(\beta_k h/2)},$$

where $\beta_k > 0$ are the roots of the equation $\beta_k \tan(\beta_k h/2) = \mu/\lambda$, $A(\beta) = 1/[\mu\lambda + (\lambda^2\beta^2 + \mu^2) \cdot h/2] \cdot \beta$.

In the balance condition (5) for each cell D_j it is necessary to take into account the heat flux through the portion $S_{\delta j}$ of the reflector back surface S_δ , which serves as a bottom for this cell, and the heat fluxes through the side walls of the cell. In particular, for the central cell D_1 , whose symmetry axis coincides with the axis Oz (see Fig. 1), we may write

$$\begin{aligned} M_1 c (T_m - T_{in}) + \lambda l \sum_{i=1}^m \int_{\delta}^{\delta+H} \frac{\partial}{\partial x} v_{1i}(x, z)|_{x=h/2} dz + \\ + \lambda \iint_{S_{\delta 1}} \frac{\partial}{\partial z} v(r, z)|_{z=\delta} dS = 0, \end{aligned} \quad (8)$$

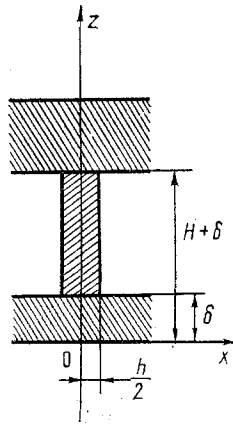


Fig. 2. Diagram of the cell wall.

where m is the number of the side walls of the cell. Assuming that $r = 0$ in Eq. (6) and using Eq. (7), we obtain

$$\lambda \sum_{n=1}^{\infty} A_n B(\alpha_n) = T_{\max} - T_m,$$

where $B(\alpha_n) = 1/[\mu \cosh(\alpha_n \delta) + \lambda \alpha_n \sinh(\alpha_n \delta)]$, $T_{\max} = T(0, \delta)$ is the maximum temperature on the cooled reflector surface.

Upon calculating the derivatives and integrals in Eq. (8) and excluding T_m in the case of cells having the shape of hexahedral prisms, we arrive at the equality

$$T_{\max} - T_{\text{in}} = \sum_{n=1}^{\infty} \lambda A_n B(\alpha_n) + \frac{6\mu\lambda}{M_1 c} \left\{ \frac{2l\mu\lambda}{S(P_{1i})} \sum_{k,n=1}^{\infty} A_n C_{n1} A(\beta_k) B(\alpha_n) \text{th}(\beta_k H) + \sum_{n=1}^{\infty} A_n C_{n2} B(\alpha_n) \right\}. \quad (9)$$

Here $l = D_h \sqrt{3}/6$,

$$C_{n1} = \iint_{P_{1i}} J_0(\alpha_n r) r dr d\varphi = 2 \int_0^{\pi/6} \int_{a_1}^{a_2} J_0\left(\frac{\alpha_n u}{\cos \varphi}\right) \frac{u du d\varphi}{\cos^2 \varphi},$$

$$C_{n2} = \iint_{P_{1i}} J_0(\alpha_n r) r dr d\varphi = 2 \int_0^{\pi/6} \int_0^{a_1} J_0\left(\frac{\alpha_n u}{\cos \varphi}\right) \frac{u du d\varphi}{\cos^2 \varphi},$$

$$a_1 = 0,5D_r, \quad a_2 = a_1 + h; \quad S(P_{1i}) = S(P_{11}) = \sqrt{3}(D_r + h)h/6, \quad i = \overline{2, 6};$$

P_{1i} is the end face of one of the cell walls D_i ; P'_{1i} is a perfect triangle amounting to the 1/6th of the cell bottom D_1 .

Having calculated the mean integral value of the heat flux through the cooled surface of the reflector

$$\bar{q} = \frac{2}{R^2} \int_0^R \left(-\lambda \frac{\partial T(r, z)}{\partial z} \Big|_{z=\delta} \right) r dr = \frac{2\mu\lambda}{R^2} \sum_{n=1}^{\infty} A_n B(\alpha_n) \int_0^R J_0(\alpha_n r) r dr = \frac{Q_0}{R^2 k_0} [1 - \exp(-k_0 R^2)], \quad (10)$$

it is possible to determine, with the aid of Eq. (9), the thermal head coefficient (the heat releasing capability of the heat exchanger) $A = \bar{q}/(T_{\max} - T_{\text{in}})$:

$$\frac{1}{A} = \frac{2k_0}{1 - \exp(-k_0 R^2)} \left\{ \sum_{n=1}^{\infty} B_n B(\alpha_n) + \right.$$

$$+ \frac{6\mu}{M_1 c} \left[\frac{2l\mu\lambda}{S(P_{11})} \sum_{k, n=1}^{\infty} B_n C_{n1} A(\beta_k) B(\alpha_n) \operatorname{th}(\beta_k H) + \sum_{n=1}^{\infty} B_n C_{n2} B(\alpha_n) \right], \quad (11)$$

where $B_n = [J_0(\alpha_n R)]^{-2} \int_0^R r \exp(-k_0 r^2) J_0(\alpha_n r) dr$.

Assuming that the heat-transfer agent velocity v_{noz} near the throats of all the nozzles is the same and that the radii of the nozzles r_j may be different, we write the heat balance condition (5) for the cell D_j in the form $M_j c(T_m - T_{\text{in}}) = \mu \lambda Q_j$, where $M_j = \pi r_1^2 \rho v_{\text{noz}}$;

$$Q_j = 2l\mu \sum_{i=1}^6 \sum_{k=1}^{\infty} A(\beta_k) \bar{v}_{ji} \operatorname{th}(\beta_k H) + \sum_{n=1}^{\infty} A_n B(\alpha_n) \iint_{P_{ji}} J_0(\alpha_n r) r dr d\varphi.$$

Then the requirement of the equality of the mean heat-transfer agent temperature in all the cells ($T_m = \text{const}$) leads to the condition

$$r_j^2 / r_1^2 = Q_j / Q_1, \quad (12)$$

which is valid for all $j = 2, \dots, p$, where p is the number of the cells; for cells equidistant from the axis Oz (Fig. 1) the radii of the nozzles are all equal, since the values of Q_j are equal for them.

Integrating equality (6) with respect to the cooled surface of the reflector S_δ , we obtain the relation

$$\bar{T} - T_m = \frac{Q_0}{\mu R^2 k_0} [1 - \exp(-k_0 R^2)], \quad (13)$$

where $\bar{T} = \int_{S_\delta} T dS / (\pi R^2)$.

Now we can determine the other most important characteristic of the considered cooling system, i.e., the effective heat-transfer coefficient $\alpha = \bar{q} / (\bar{T} - T_{\text{in}})$. Having added equalities (5) with respect to all $j = 1, \dots, p$ and taking into account Eqs. (10) and (13), we find

$$\frac{1}{\alpha} = \frac{1}{\mu} - \frac{\lambda}{M c \bar{q}} \iint_S \frac{\partial T}{\partial n} dS. \quad (14)$$

Here $S = \bigcup_{j=1}^p S_j$ is the cooled surface of all the cells, $M = \sum_{j=1}^p M_j = \pi \rho v_{\text{noz}} \sum_{j=1}^p r_j^2$ is the overall heat-transfer agent flow rate.

The integral on the right-hand side of Eq. (14) can be presented in the form

$$- \frac{\lambda}{M c \bar{q}} \iint_S \frac{\partial T}{\partial n} dS = \frac{\mu \lambda (Q_1 + \sum_k \sum_{i_k} Q_{i_k})}{\pi c \rho v_{\text{noz}} (r_1^2 + \sum_k \sum_{i_k} r_{i_k}^2)}.$$

Here, each sum $\sum_{i_k} Q_{i_k}$ is the overall flux through the cooled surfaces of all the cells which are equidistant from the axis Oz passing through the center of the mirror surface. The radii of the nozzles of these cells enter into the sum $\sum_{i_k} r_{i_k}^2$. Further, using relation (12), we can write

$$\frac{Q_1 + \sum_k \sum_{i_k} Q_{i_k}}{r_1^2 + \sum_k \sum_{i_k} r_{i_k}^2} = \frac{(Q_1 + \sum_k \sum_{i_k} Q_{i_k}) Q_1}{r_1^2 (Q_1 + \sum_k \sum_{i_k} Q_{i_k})} = \frac{Q_1}{r_1^2}.$$

With account of Eq. (11), we finally obtain

$$\frac{1}{\alpha} = \frac{1}{A} + \frac{1}{\mu} - \frac{2k_0}{1 - \exp(-k_0 R^2)} \sum_{n=1}^{\infty} B_n B(\alpha_n). \quad (15)$$

In the case of a uniform distribution of the heat flux $q(r)$ over the mirror surface (uniform illumination), when $k_0 = 0$, $q(r) = Q_0$, the radii of all the nozzles are identical ($r_j = r_1$, $M_j = M_1$, $j = 2, \dots, p$); $B_n = 0$, $n \geq 2$; $B_1 = R^2/2$, $B(\alpha_1) = 1/\mu$, then relations (11) and (15) yield

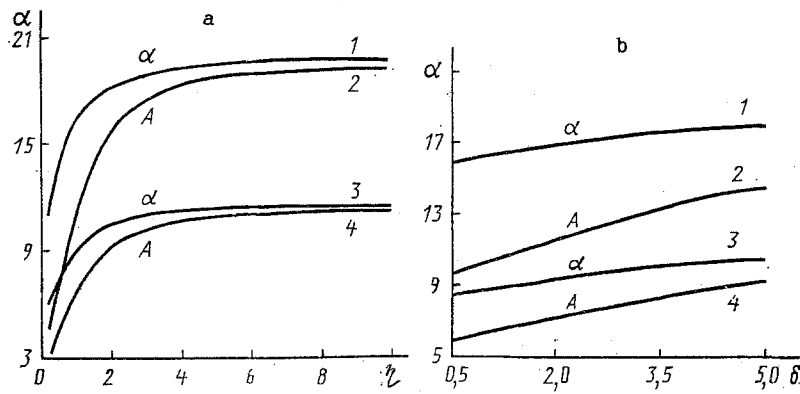


Fig. 3. Effect of the relative irradiation radius of the mirror surface $\eta = r_0/R$ (a) and of the reflector thickness δ , mm (b) on the effective heat-transfer coefficient α , kW/(m²·K) and thermal head coefficient A , kW/(m²·K), at $\delta = 1$ mm (a); $\eta = 1$ (b), $v_{noz} = 1$ m/sec for different hydraulic diameters of water-cooled cells D_h : 1, 2) $D_h = 3.25$ mm; 3, 4) 6.5 mm.

$$\frac{1}{\alpha} = \frac{1}{A} = \frac{1}{\mu} + \frac{V\sqrt{3}D_h}{2M_1c} \left[8\mu\lambda \sum_{h=1}^{\infty} A(\beta_h) \text{th}(\beta_h H) + D_h \right]. \quad (16)$$

Note that for cells having the shape of tetrahedral prisms (the basis is a square), one can use the obtained formulae but with the following changes: it is necessary to discard the factor $\sqrt{3/2}$ in Eq. (16) and to replace the factor 6 by 4 in Eq. (11). When determining the constants C_{n1} and C_{n2} , the outward integral should be calculated for the interval $[0, \pi/4]$; it is also necessary to take into account that $S(P_{11}) = 0.5h(D_h + h)$, $l = 0.5D_h$.

Comparing Eq. (16) with the corresponding formula for the case of tetrahedral cells, we come to the conclusion that in the case of the fixed hydraulic diameter of the cells and other parameters being equal (except l), the heat removing ability of the cooling system with hexahedral cells is higher. This is attributable to a larger area of the cooled surface of the cell in the case of tetrahedral cells with the same heat-transfer agent flow rate per one nozzle.

For the same reason, the rise in the height of the cell walls leads to a decrease in α and A , as can be easily demonstrated with the help of Eqs. (11) and (16) taking into account the fact that $f(H) = \text{th}(\beta H)$ is a monotonically increasing function.

The expressions obtained for the effective heat-transfer coefficient and the thermal head coefficient indicate that the basic reserve for heat-transfer enhancement is the heat-transfer coefficient μ on the surfaces that contact the heat-transfer agent. There are several possibilities for its increase (see Eq. (4)): 1) by decreasing the hydraulic diameter of the cell; 2) by increasing the heat-transfer agent velocity; 3) by applying heat-transfer agents with a high thermal conductivity and low viscosity. Increasing μ infinitely in Eqs. (11), (15), and (16), we arrive at the physical limits for the considered characteristics of cooling systems:

a) with hexahedral cells

$$\alpha = A = \frac{[1 - \exp(-k_0 R^2)] M_1 c}{12k_0 \sum_{n=1}^{\infty} B_n C_{n2} \text{sech}(\alpha_n \delta)}; \quad (17)$$

b) with tetrahedral cells

$$\alpha = A = \frac{[1 - \exp(-k_0 R^2)] M_1 c}{8k_0 \sum_{n=1}^{\infty} B_n C_{n2} \text{sech}(\alpha_n \delta)}. \quad (18)$$

For a uniformly illuminated mirror we have, respectively,

$$\alpha = A = 2\sqrt{3}M_1c/3D_h^2 \quad \text{and} \quad \alpha = A = M_1c/D_h^2. \quad (19)$$

For a copper mirror with water-cooled hexahedral cells the values of α and A were calculated from Eqs. (11) and (15)-(19) with the following values of the parameters: $\lambda = 380$ W/(m·K), $\lambda_l = 0.67$ W/(m·K), $c = 4186$ J/(kg·K), $\rho = 965$ kg/m³, $\nu = 0.326 \cdot 10^{-6}$ m²/sec, $R = 50$ mm, $H = 5$ mm, $h = 1$ mm, $r_1 = 0.5$ mm. Some of the results of the calculations are illustrated in Figs. 3 and 4.

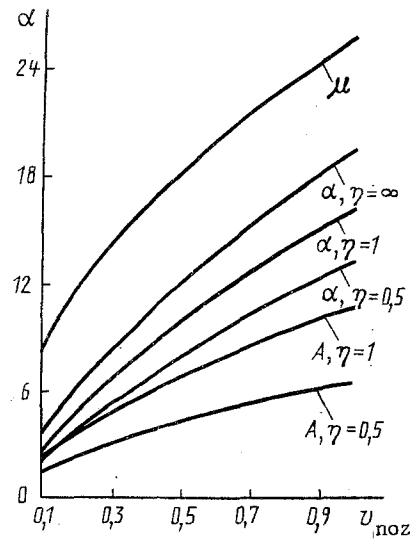


Fig. 4. Effect of the heat-transfer agent velocity v_{noz} m/sec at different relative irradiation radii of the mirror surface η on the heat-transfer coefficient of the surfaces of the cell walls μ , kW/(m²·K), effective heat-transfer coefficient α , kW/(m²·K) and thermal head coefficient A , kW/(m²·K) at $D_h = 3.25$ mm; $\delta = 1$ mm.

Analyzing the graphical relations obtained, we arrive at the following conclusions.

The increase of the homogeneity of the mirror surface illumination, i.e., the increase in the relative illumination radius $\eta = r_0/R$, leads to the growth of α and A (Fig. 3a) with $\alpha > A$, and the range of variation of the thermal head coefficient is much wider ($\alpha \gg A$ at small η 's). With an infinite increase in η (uniform illumination), $\alpha = A$.

The increase of the reflector thickness ($0.5 \text{ mm} < \delta < 5 \text{ mm}$) leads to a slight growth of α and A (Fig. 3b), which is due to the temperature fall on the cooled reflector surface; however, a further rise of δ will lead to an undesirable growth of temperature and thermal stresses on the surface being irradiated.

From Fig. 4 it follows that $\alpha < \mu$, i.e., the effective heat-transfer coefficient is smaller than the heat-transfer coefficient on the surfaces of the cell walls. This is due to the fact that heat transfer through the bottom surfaces of the cells exceeds that through the bottoms of the side walls.

It is possible to attain a substantial increase in the values of the heat-transfer characteristics α and A by decreasing the hydraulic diameter of the cells D_h down to several millimeters (see Fig. 3) and also by increasing the heat-transfer agent velocity v_{noz} (i.e., by increasing its flow rate), Fig. 4. It should be noted here that the reduction of the hydraulic diameter of the cell increases the resistance to liquid flow, thus imposing restrictions on the velocity because of the mechanical action of the jet on the reflector. This circumstance should be taken into account when developing a specific scheme of heat removal.

Comparison of the results of calculations presented in Figs. 3 and 4 with those given in [5] for the effective heat transfer in a jet heat exchanger having cylindrically shaped cells shows that the data from [5] (Tables 3 and 4) are about two times those we obtained for α . It can be assumed that a certain disagreement of the results is due to the fact that the values of α given in [5] were obtained for small distances between the nozzle tip and the cell bottom ($h_{noz}/d_{noz} < 1$) when the values of the function $f(h_{noz}/d_{noz})$, which influences heat transfer in the cell (see Eq. (3)), exceed 1.

Calculations by Eqs. (17)-(19) show that a considerable increase in heat transfer from the surfaces of the cell walls for the nozzle Reynolds numbers of order 10^4 - 10^5 in a copper heat exchanger could have resulted in the effective heat transfer flux from 300 to 3000 kW/(m²·K).

NOTATION

R , mirror radius; δ , reflector thickness; r_0 , radius of laser beam incident on mirror; $k_0 = 2r_0^{-2}$; $\eta = r_0/R$, relative radius of irradiation; T , T_m , T_{in} , temperature, mean temperature of heat-transfer agent, heat-transfer agent temperature at nozzle inlet;

$v = T - T_m$; D_h , hydraulic diameter of cell; l , h , H , length, thickness and height of cell wall, respectively; r_j , radius of cell nozzle D_j ; v_{noz} , heat-transfer agent velocity at nozzle exit; h_{noz} , distance between nozzle tip and cell bottom; d_{noz} , nozzle diameter; $q(r)$ density of heat flux absorbed by mirror surface; Q_0 , heat flux density at mirror center; α , effective heat-transfer coefficient; A , thermal head coefficient; λ , thermal conductivity coefficient of mirror material; λ_t , thermal conductivity coefficient of heat-transfer agent; c , ρ , specific heat and density of heat-transfer agent; ν , kinematic viscosity of heat-transfer agent; M_j , heat-transfer agent flow rate in cell D_j ; S_j , cooled cell surface D_j ; S_δ , cooled surface of reflector; $S_{\delta j}$, bottom surface of cell D_j ; p , number of cells; J_0 , J_1 , Bessel functions; \sinh , \cosh , \tan , sech , hyperbolic sine, cosine, tangent, and secant, respectively; Re , Nu , Pr , Reynolds, Nusselt and Prandtl number, respectively; μ , coefficient of heat transfer from the surfaces of cell walls.

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